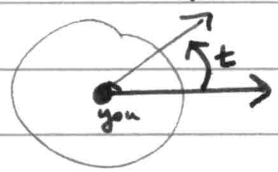
1060 – Class 13
Fall 2013

4.2 - Measuring Angles

Imagine you are in a field,
facing East,
and that you rotate by some angle t



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Amount of rotation $= \dfrac{\text{angle rotated}}{\text{total } \measuredangle\text{'s in a circle}}$

360° in a circle $\Rightarrow$ 90° rotation is $\frac{1}{4}$ of a full rotation

2π radians in a circle $\Rightarrow$ $\frac{\pi}{2}$ is $\frac{1}{4} \cdot 2\pi$

So $\frac{\pi}{2}$ is $\frac{1}{4}$ of a full rotation

That is: $\frac{\pi}{2}$ radians $= 90° = \frac{1}{4}$ of a full rotation

<u>Because</u> 1 circle = 2π radians = 360 degrees

$$1 \text{ radian} = \frac{180^\circ}{\pi \text{ radians}}$$

and

$$1^\circ = \frac{\pi \text{ radians}}{180^\circ}$$

<u>Eg:</u> Write 45° in Radians

$$45^\circ \cdot \left(\frac{\pi \text{ radians}}{180^\circ}\right) = \frac{45}{180} \cdot \pi \text{ radians}$$

$$\boxed{45^\circ = \frac{\pi}{4} \text{ radians}}$$

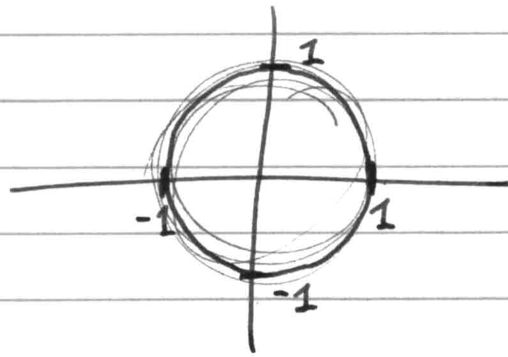
<u>Eg:</u> Write $\frac{\pi}{3}$ radians in degrees

$$\frac{\pi}{3} \cdot \left(\frac{180^\circ}{\pi}\right) = \frac{\pi}{\pi} \cdot \frac{180^\circ}{3} = 60^\circ$$

$$\boxed{\frac{\pi}{3} \text{ radians} = 60^\circ}$$

The <u>unit circle</u>

is the set of all points
distance = 1 from the point (0,0)



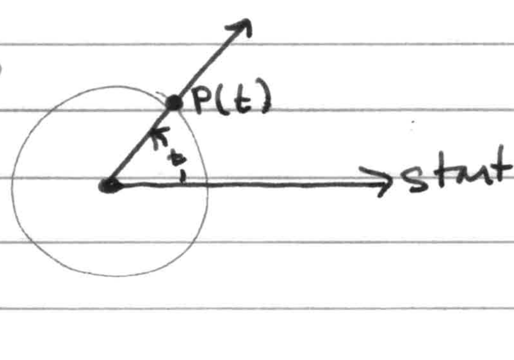
it is <u>Nice</u> to think about ∡'s using the circle

- our default starting direction is the positive x-axis
- Positive #'s mean <u>COUNTERCLOCKWISE</u> movement

for each number t,

let P(t) be the point where
the ray making ∡ t with the positive x-axis
hits the unit Circle

<u>That is:</u> If $t > 0$

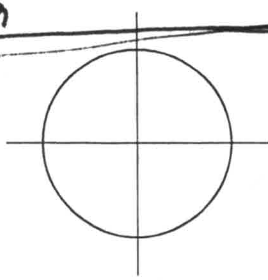


(1) Do all of the recommended homework in the textbook.

(2) Plot the approximate location of the following angles on the unit circle.

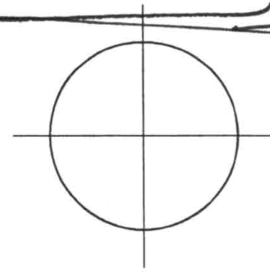
Did in class

(a) $\theta = \frac{3\pi}{4}$ radians

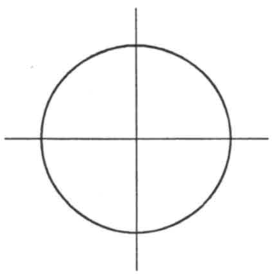


(b) $\theta = -\frac{\pi}{4}$ radians

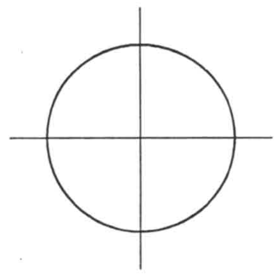
8) $\theta = \frac{11\pi}{6}$ radians



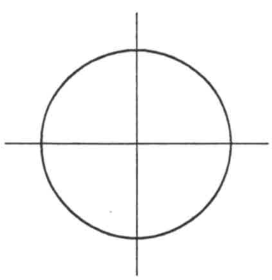
(c) $\theta = -\frac{11\pi}{6}$ radians



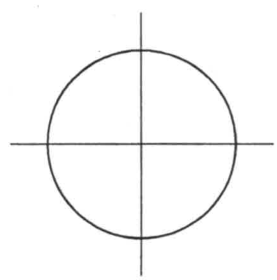
(d) $\theta = \frac{2\pi}{3}$ radians



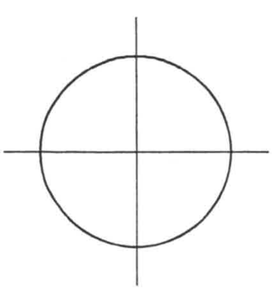
(e) $\theta = \frac{9\pi}{2}$ radians



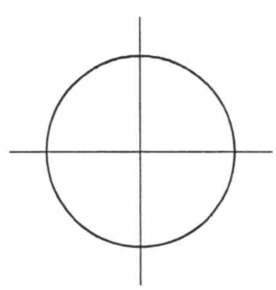
(f) $\theta = 8\pi$ radians



(g) $\theta = 45$ degrees



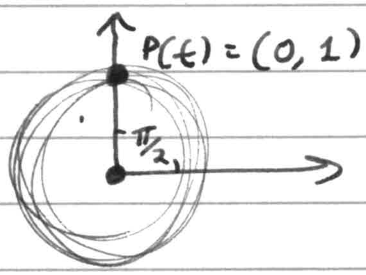
(h) $\theta = -120$ degrees.



More **Eg's**: for each of the angles t below, plot $P(t)$

Ⓐ $t = \frac{\pi}{2}$

Remember $\frac{\pi}{2}$ is $\frac{1}{4}$ of a full circle



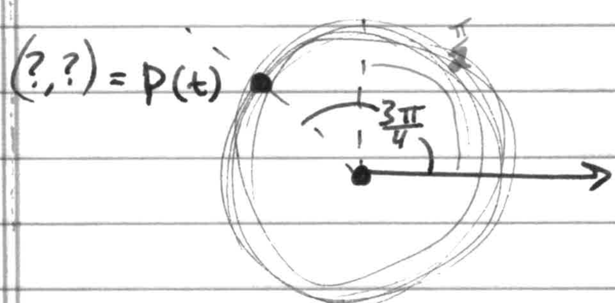
5 - DO FAST

Ⓑ $t = \frac{3\pi}{4}$ $\left(\text{Think: } \text{[illegible]} = \frac{3}{8} \text{ of a circle} \right)$

$\frac{1}{4}$ circle + $\frac{1}{8}$ circle

$= \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4}$

$= \frac{1}{4}$ circle + $\frac{1}{2}$ of $\frac{1}{4}$ circle



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lots more eg's on the homework...

DO Them for Practice!

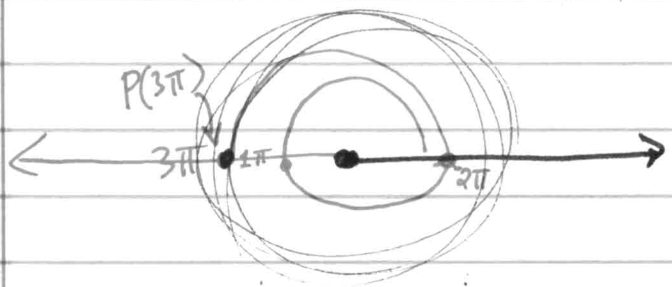
we often use greek letters for angles

especially $\theta = \theta$

Ⓒ $\theta = 3\pi$

(Think: 3π is $\frac{3}{2} \cdot 2\pi$

is $\frac{3}{2}$ times around the whole circle)

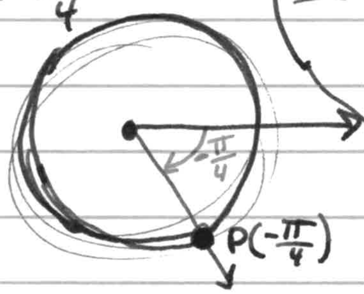


Notice: $\boxed{\text{$3\pi$ and π give the same x.}}$

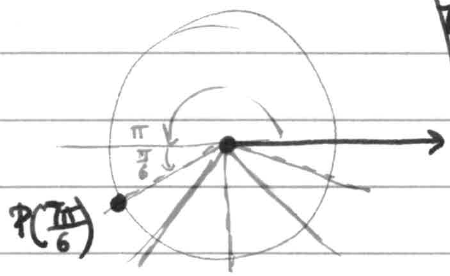
Ⓓ $\theta = \frac{-\pi}{4}$

(Think: $-\frac{\pi}{4}$ is $-\frac{1}{8} \cdot 2\pi$

so $\frac{1}{8}$ of a circle, in the CLOCKWISE direction)



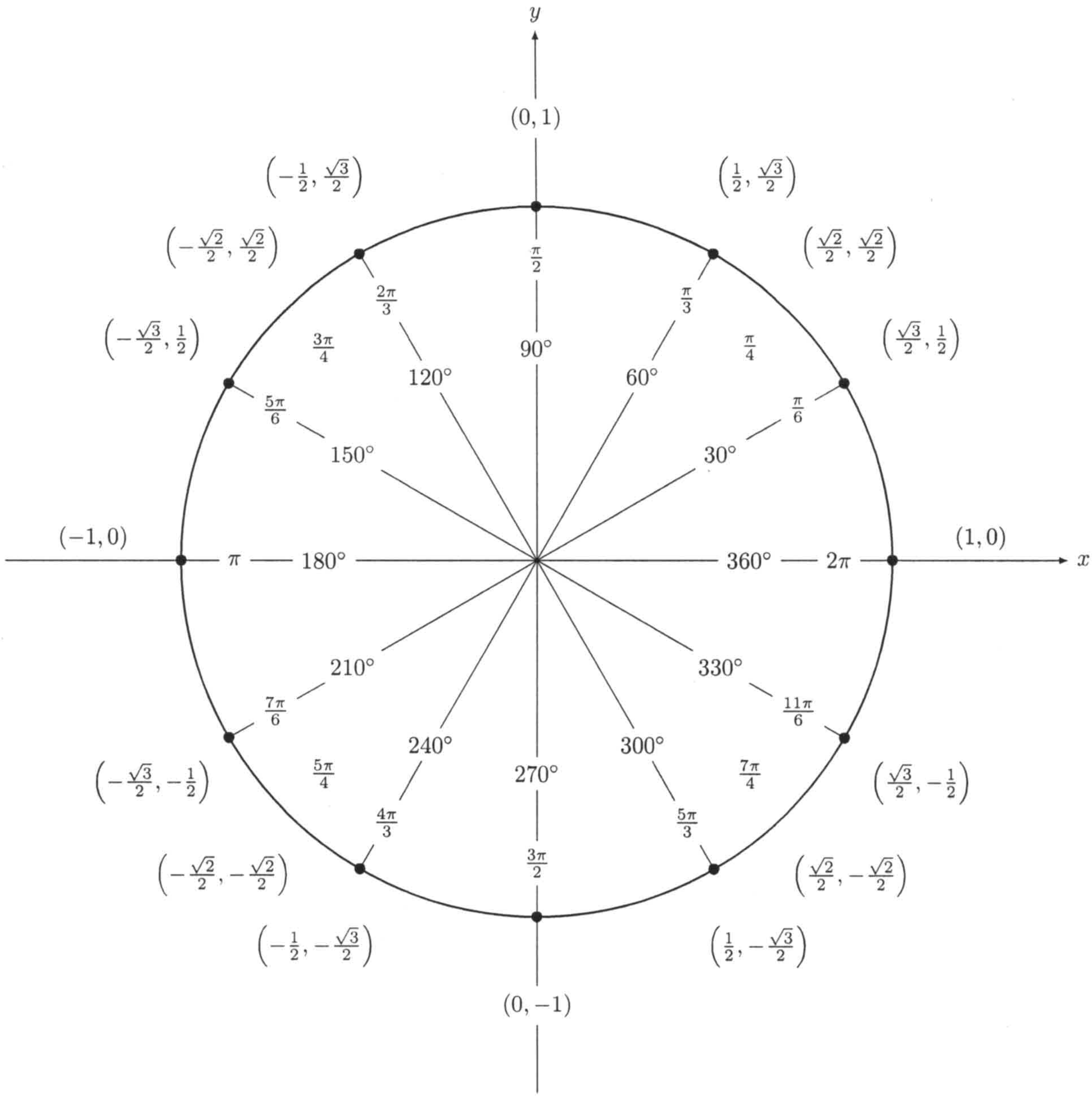
Eg: Sketch $P\left(\frac{7\pi}{6}\right)$



Think: $\frac{7\pi}{6}$ is $\frac{1\pi}{6} + \frac{6\pi}{6}$

which is $\frac{\pi}{6} + \pi$

this is π followed by $\frac{\pi}{6}$



y
x
$(0,1)$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$(-1,0)$
$(1,0)$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$(0,-1)$
$\frac{\pi}{2}$ 90°
$\frac{2\pi}{3}$ 120°
$\frac{3\pi}{4}$
$\frac{\pi}{3}$ 60°
$\frac{\pi}{4}$
$\frac{5\pi}{6}$ 150°
$\frac{\pi}{6}$ 30°
π 180°
360° 2π
210° $\frac{7\pi}{6}$
330° $\frac{11\pi}{6}$
240° $\frac{5\pi}{4}$ $\frac{4\pi}{3}$
270° $\frac{3\pi}{2}$
300° $\frac{7\pi}{4}$ $\frac{5\pi}{3}$

Eg:

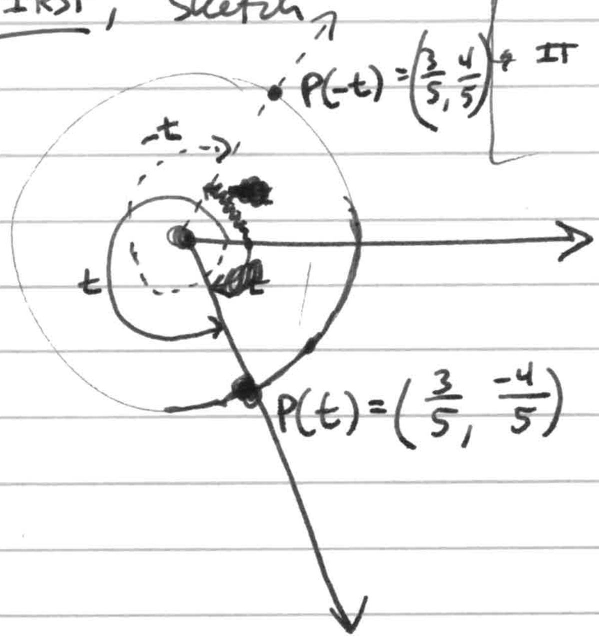
If we know $P(t) = \left(\frac{3}{5}, \frac{-4}{5}\right)$

find $P(-t)$ and $P(t+\pi)$

FIRST, Sketch

t could either $\boxed{\text{be the big } \measuredangle \text{ if } t>0}$
or $\boxed{\text{the small } \measuredangle \text{ if } t<0}$
& IT DOESN'T MATTER - pick what makes you happy

10
-15



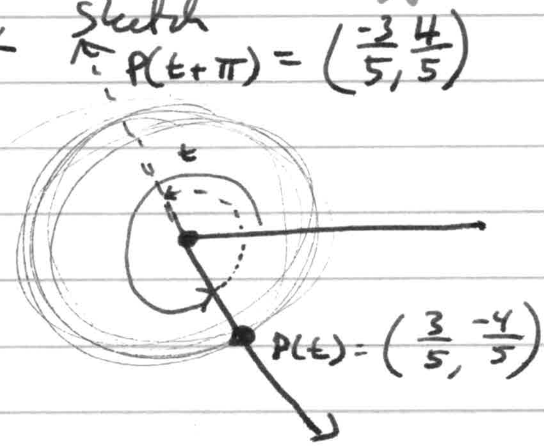
$P(-t) = \left(\frac{3}{5}, \frac{4}{5}\right)$

still positive

Now positive

Both signs flip

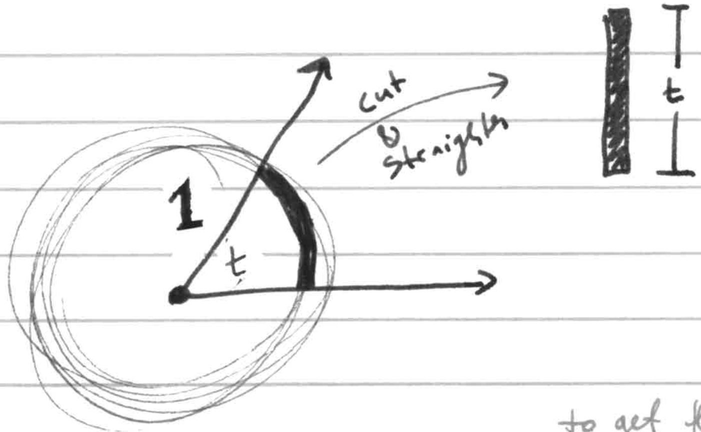
Again, Sketch



$\Rightarrow \quad P(t+\pi) = \left(\frac{-3}{5}, \frac{4}{5}\right)$

Arc length:

Radian angle measure
has a neat property



Cut the unit circle to get the arc with $\measuredangle t$

The length of this arc is equal to t

More generally: if a circle has radius r,
the arc made by the angle t
has length

$$\left(\text{length of arc}\right) = (\text{radius}) \cdot (\text{angle})$$

aka. arc-length

ONLY works for Radians!

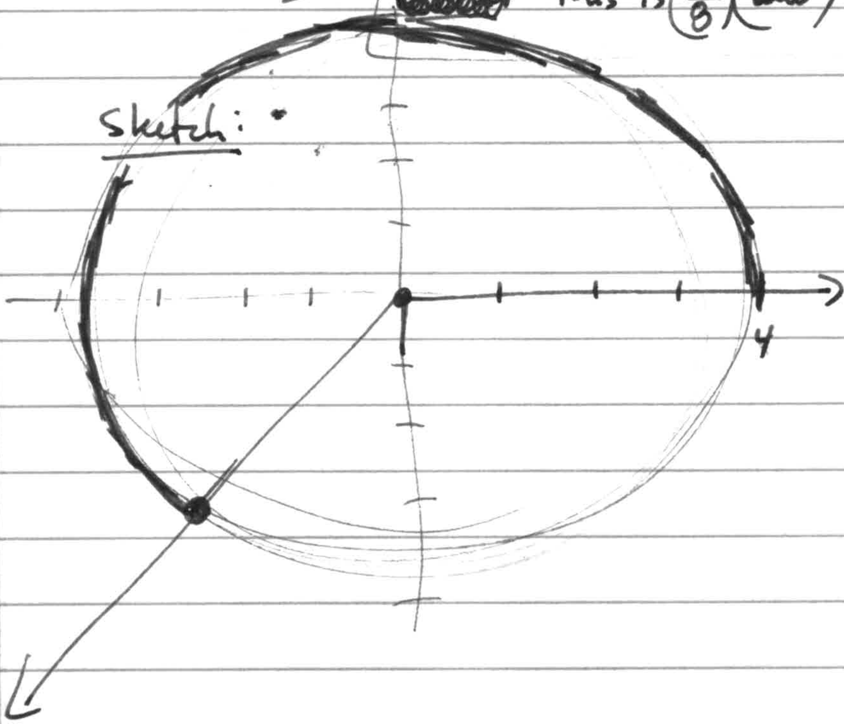
Eg: fix the circle with radius 4"

find the length of the arc intercepted by the angle

$\theta = \frac{5\pi}{4}$ $\boxed{\text{radians}}$

Notice: $\boxed{\text{Remember: this is } \left(\frac{5}{8}\right)(2\pi) = \left(\frac{5}{8} \text{ of a circle}\right)}$

Sketch:

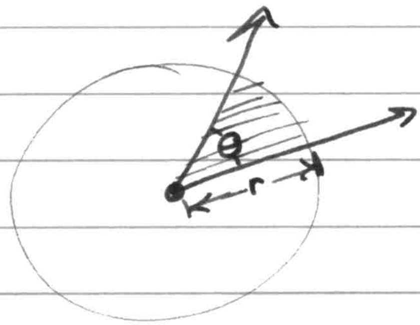


arc-length = (radius)·(arc-length in radians)

$\boxed{\text{If you are given } \angle \text{ in degrees, } \boxed{\text{you must convert to radians first}}}$

$\boxed{\text{Most HW problems are like this!}}$

Area of a Circle's sector



$$\left(\text{area of sector}\right) = \frac{1}{2}\cdot r^2 \cdot \theta$$

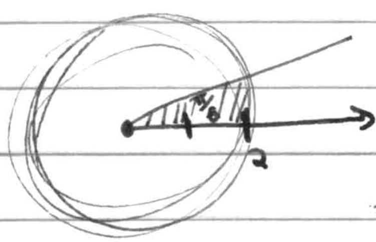
$\boxed{\text{ONLY works when } \theta \text{ is in Radians!}}$

Eg:

Find the area A of a sector with angle 30° in a circle of radius 2.

$$30^\circ \cdot \left(\frac{\pi \text{ radians}}{180^\circ}\right) = \frac{30}{180}\cdot\pi \text{ radians}$$

$$= \frac{\pi}{6}\text{ radians}$$



$$\left(\begin{array}{c}\text{area}\\ \text{of sector}\end{array}\right) = \frac{1}{2}(2)^2 \cdot \frac{\pi}{6}$$

$$= \frac{4}{12}\pi = \frac{\pi}{3}$$

$\boxed{\text{area of sector}}$